

### MINI LECTURE – CONCENTRATION ELECTROCHEMICAL CELLS

**Cells are remarkably complex systems. However, we can understand some key characteristics and functions of cells from simple models. One of the most remarkable aspects of many cells is that they utilize energy to maintain different concentrations of ions inside the cell compared to the solution outside. This difference in ions then creates an electrical potential that the cells can use for signaling (and which can be controlled via ion concentrations).**

For a typical muscle cell in your heart there are three key ions that have concentrations differences inside and outside of the cell:  $\text{Na}^+$ ,  $\text{K}^+$ , and  $\text{Ca}^{2+}$ . Let's look at the simplest situation of just the  $\text{K}^+$  ions. The concentration of potassium ion inside the cell is higher than outside:  $[\text{K}^+]_{\text{in}} > [\text{K}^+]_{\text{out}}$ . Therefore  $\text{K}^+$  should be constantly diffusing out of the cell. However, the negative counter ions (chloride, proteins,...) cannot pass through the membrane. This means that as the  $\text{K}^+$  diffuses out a negative charge builds up in the cell. At some point, equilibrium is reached where the driving force for the outward diffusion (the concentration difference) is balanced by the electric force from the potential difference. This is the "rest potential" of the cell. We can determine the rest potential for the  $\text{K}^+$  from the Nernst equation.

The "reaction" is simply the diffusion the  $\text{K}^+$



1. For this "reaction", what is the reaction quotient Q?

$$Q = \frac{K^+_{\text{out}}}{K^+_{\text{in}}}$$

2. For this "reaction" what is the std potential?

Since the two half reactions are the same AND the concentrations of the ions under standard conditions are both 1M (aka the same concentrations) then, the standard potential is zero. Nobody "wins" the free energy game. That is, there is NO potential difference.

3. Using the Nernst equation, write an expression for the potential based on the concentrations inside and outside the cell. (note: for physiological conditions the constant in the Nernst equation is slightly larger as the temperature is higher).

We can use the room temperature Nernst Equation:

$$E = E^o - \frac{RT}{nF} \ln Q$$

$$E = -\frac{RT}{nF} \ln \frac{[K^+_{out}]}{[K^+_{in}]}$$

$$E = -\frac{0.0591}{n} \log \frac{[K^+_{out}]}{[K^+_{in}]}$$

Alternatively, we can adjust the Nernst Equation to account for body temperature which is 37°C or 310K:

$$E = E^o - \frac{RT}{nF} \ln Q$$

$$E = -\frac{RT}{nF} \ln \frac{[K^+_{out}]}{[K^+_{in}]}$$

OR

$$E = E^o - \frac{8.314 \frac{J}{molK} \times 310K}{n \times 96485 \frac{C}{mole^-}} \times 2.3 \times \log \frac{[K^+_{out}]}{[K^+_{in}]} \quad (2.3 \text{ is the conversion between ln and log})$$

$$E = E^o - \frac{0.0614}{n} \log \frac{[K^+_{out}]}{[K^+_{in}]}$$

$$E = -\frac{0.0614}{n} \log \frac{[K^+_{out}]}{[K^+_{in}]}$$

4. How large a potential can you build up (polarize)? It depends on the concentration difference. Given the typical concentrations for a muscle cell in your heart are  $[K^+]_{in} = 150 \text{ mM}$  ;  $[K^+]_{out} = 4 \text{ mM}$ , what is this potential?

$$E = -\frac{0.0614}{n} \log Q$$

$$E = -\frac{0.0614}{1} \log \left( \frac{4}{150} \right)$$

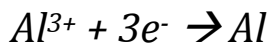
$$E = 0.0966V$$

$$E = 97mV$$

(Converting between mM and M does not matter because the units cancel out inside the logarithm expression anyway.)

## MINI LECTURE ELECTROLYTIC CELLS

**Aluminum metal is produced primarily from reduction of aluminum oxide via electrolysis. The actual process involves a number of compounds including  $\text{Na}_3\text{AlF}_6$  and  $\text{Al}_2\text{O}_3$  but the general process can be simplified to be:**

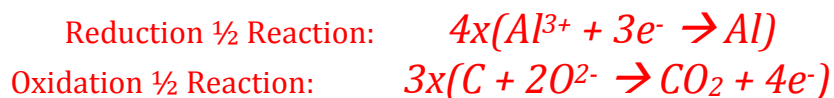


1. What is the overall balanced equation for this reaction? What reaction would occur at the anode? The cathode?

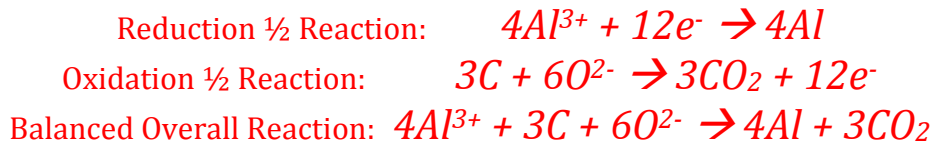
Our goal is to produce aluminum metal. So aluminum metal must be one of our products. The other reaction must include electrons on the opposite side of the equation from the aluminum reduction so the two half reactions would be:



The two half-reactions are balanced in terms of elements and charge individually. In order to combine them though, we must eliminate the electrons by multiplying the reactions with coefficients. The lowest common multiple of 3 and 4 is 12.



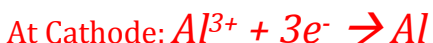
Combine:



**Oxidation ALWAYS** occurs at the **anode**. The oxidation of carbon would occur:



**Reduction ALWAYS** occurs at the **cathode**. The reduction of aluminum would occur.



2. What total charge, in Coulombs, is needed to produce 1 kg of Al ( $27 \text{ g mol}^{-1}$ )?

This is just a dimensional analysis problem:

1 kg Al	1000 g Al	1 mol Al	3 mol e <sup>-</sup>	96,485 C
	1 kg Al	27 g Al	1 mol Al	1 mol e <sup>-</sup>

= 10720555.6 C = 10.72 MC (mega Coulombs)

We could have used the balanced equation to compare moles of Al to moles of electrons in fourth column of this dimensional analysis. In the balanced equation, 4 moles of Aluminum are reduced when 12 moles of electrons are being used. However, the ratio of 4 moles Al to 12 moles electrons is the SAME ratio as 1 mole Al to 3 moles electrons. 4:12 = 1:3.

3. Assuming you wanted to produce 1 kg of Al in one hour, what current would be required?

Current, I, is measured in Amps which has units  $\frac{C}{s}$  (Coulombs per second). So the total charge would be the current multiplied by the time in seconds. One hour has 3,600 seconds (60 seconds X 60 minutes = 1 hour).

$$Q = (I)(t)$$

$$I = Q/t$$

$$I = (10720555.6 C) \div (3600 \text{ seconds}) = 2977.932 \text{ Amps}$$

4. The electrical power needed is the current times the voltage of the electrochemical cell (Amps x Volts = Watts). This reaction needs to be run with an applied potential of approximately 5 V to efficiently drive the electrolysis. For the given current, how much power would be needed to produce the 1 kg of Al?

$$\text{Power} = (I)(V) = (2977.932 \text{ Amps})(5V) = 14889.660 \text{ Watts}$$

5. Electrical energy is sold in units of kW-hr (power x time). Given that you used the above power for 1 hr, how many kW-hr of energy did you need?

$$\text{Power} = 14889.660 \text{ Watts} = 14.88966 \text{ kW}$$

$$\text{Energy} = (P)(t) = (14.88966 \text{ kW})(1\text{hr}) = 14.88966 \text{ kW-hr}$$

6. The cost of 1 kW-hr varies greatly, but assuming it is approximately \$0.10/kW-hr, how much would electricity cost alone be for the production of your 1 kg of Al?

$$\text{Cost} = (\text{Energy})(\text{Price}) = (14.88966 \text{ kW-hr})(0.10\$) = \$1.49$$

7. The commodities cost of Al is currently about \$1.50/kg. What fraction of this cost is the electricity?

Fraction of Electricity per Total Cost =  $(\$1.49)/(\$1.50) = 0.9926$  or about 99.26%

8. In the production of your 1 kg of Al, how many kg of carbon are consumed?

1 kg Al	1000 g Al	1 mol Al	3 mol C	12 g C	1 kg C
	1 kg Al	27 g Al	4 mol Al	1 mol C	1000 g C
		= 0.33 kg C			